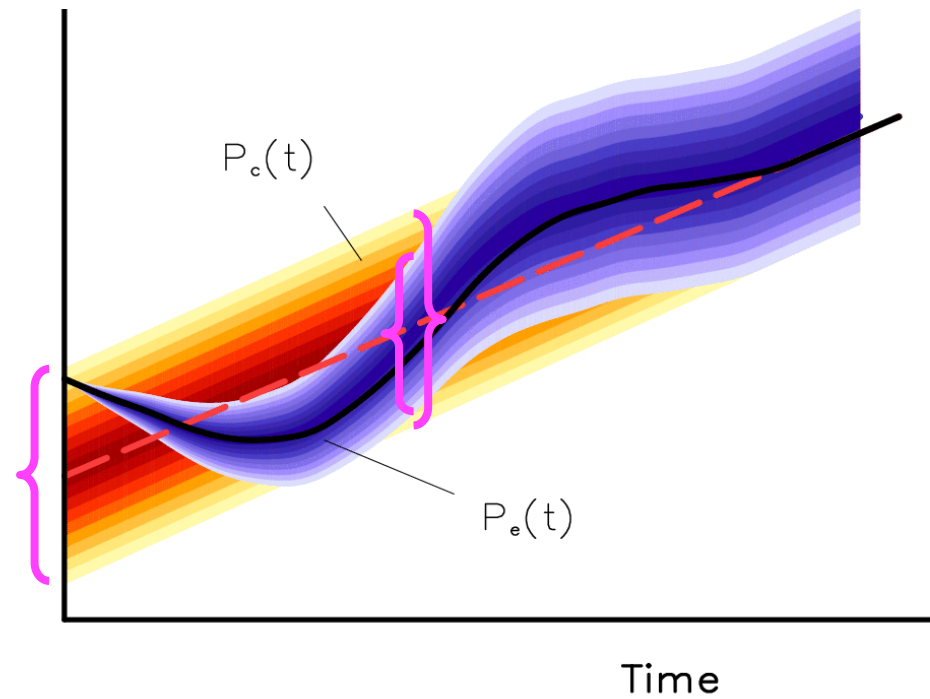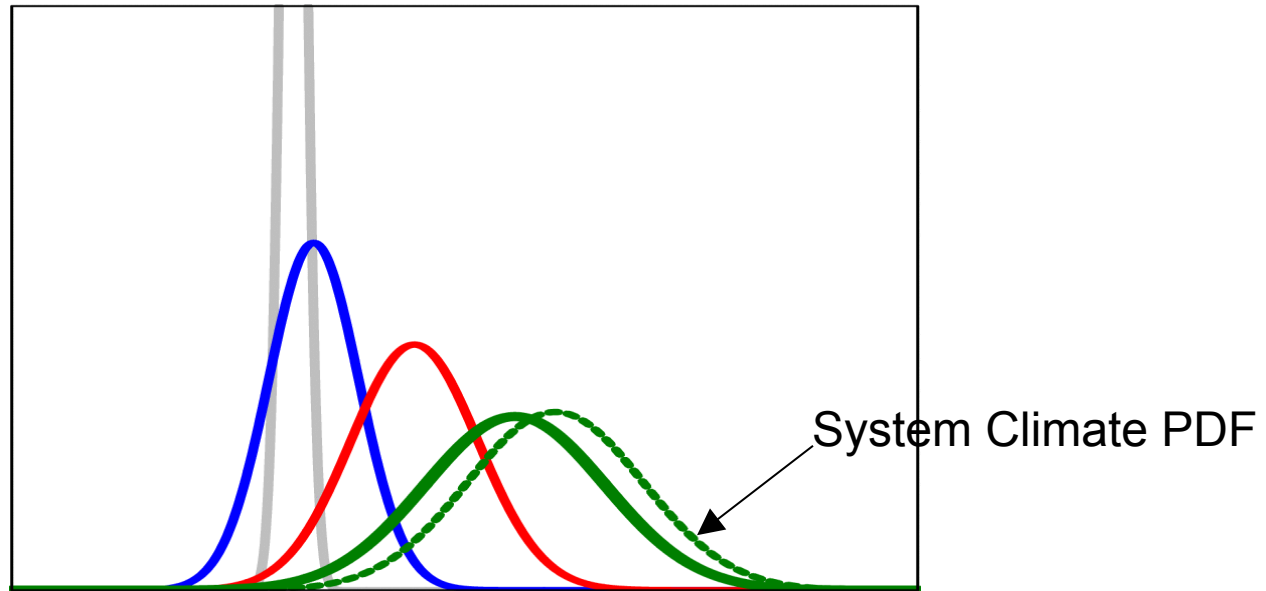# Uncertainty in Decadal Predictions
Resulting from Imperfect Knowledge of the Initial Conditions

Grant Branstator
Haiyan Teng
NCAR

Initial Value & Forced Predictability

Predictability Measures

System Climate PDF

Mean Square Error

$$MSE = \frac{1}{K}\sum_{k=1}^{K}\left[\left(\frac{1}{M}\sum_{m=1}^{M}(s_k^m - \bar{s}_k)^2 \right) / \sigma_k^2 \right]$$

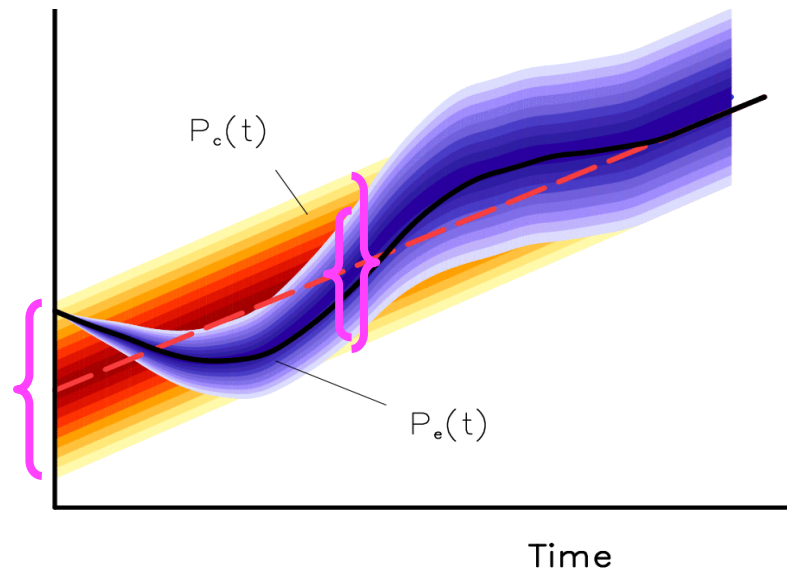where s_k^m is PC k of ensemble member m

Relative Entropy
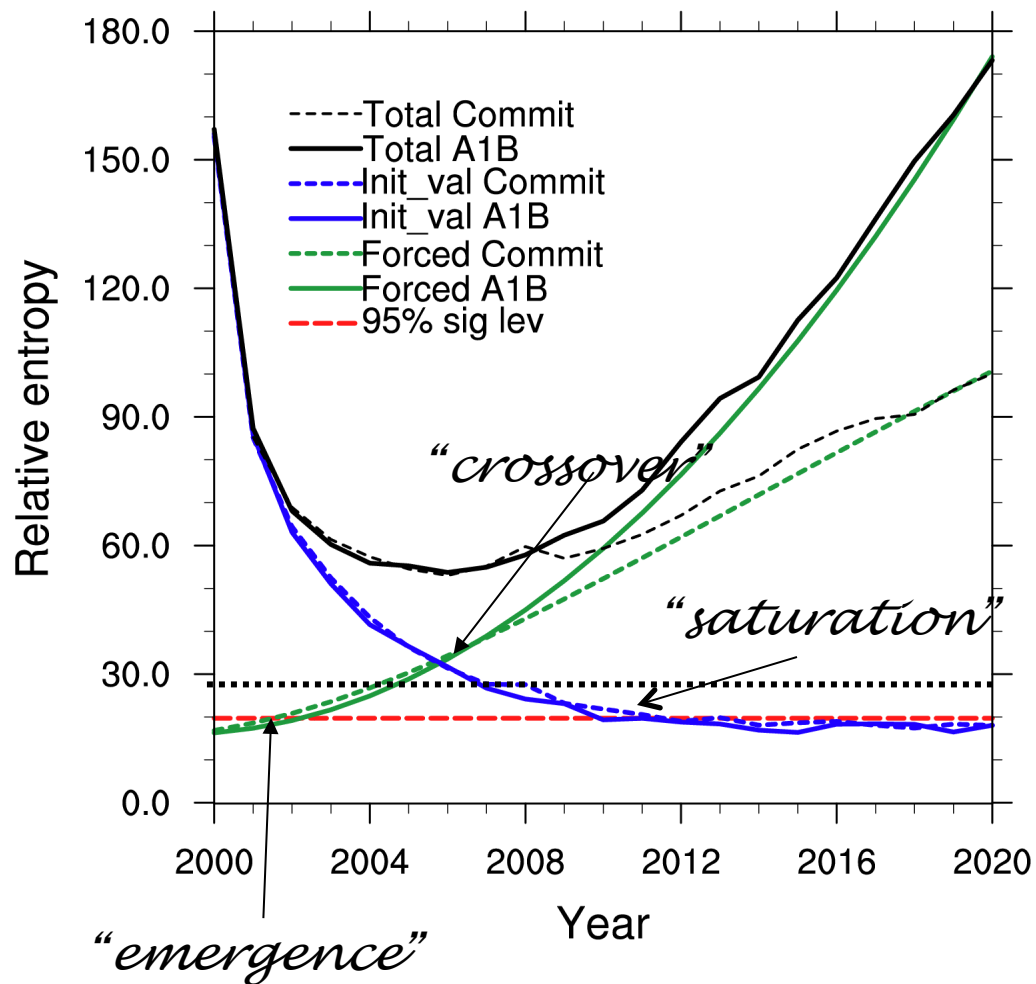
$$R = \int_S P_e(s)\ln\left[\frac{P_e(s)}{P_c(s)} \right]ds$$
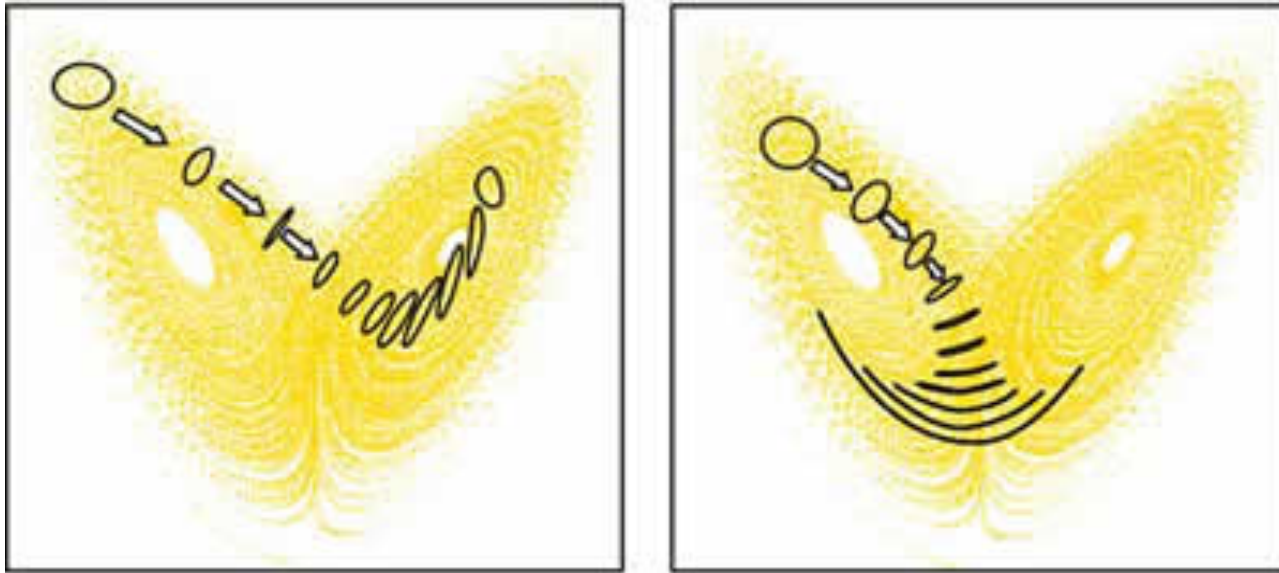
bits of information

Example

Ocean Heat Content

Global T$_{0\text{-}300m}$
CCSM3 40 member ensembles
Sum of R15 at 8 Subdomains

Relative entropy

180.0
150.0
120.0
90.0
60.0
30.0
0.0

- - - - Total Commit
——— Total A1B
- - - - Init_val Commit
——— Init_val A1B
- - - - Forced Commit
——— Forced A1B
- - - - 95% sig lev

"crossover"

"saturation"

10 bits

"emergence"

2000   2004   2008   2012   2016   2020

Year

Branstator & Teng (2010)

Need a more complete analysis of predictability

Lorenz 3-component model

Palmer, 1993

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z. \qquad \rho = 28, \ \sigma = 10, \ \beta = 8/3$$

Predictability and Power Spectra
Redness

For $\dot{s} = -\alpha s(t) + \xi(t)$ with Gaussian white noise ξ

$$\sigma_c^2 = \sigma_\xi^2 / 2\alpha$$

$$\text{cor}(\tau) = \exp(-\alpha\tau) = \exp(-\tau/T_d)$$

$$\frac{\sigma_e^2(\tau)}{\sigma_c^2} = 1 - \exp(-2\alpha\tau) = 1 - \exp(-2\tau/T_d)$$

atmos: T_d=roughly a week
OHC: T_d=couple of years

Also note that T_d increases for time averages
of such a process.
(Griffies&Bryan 1997; Munk 1960)

Predictability and Power Spectra

Average T_{0-500m} Spectrum
### North Atlantic
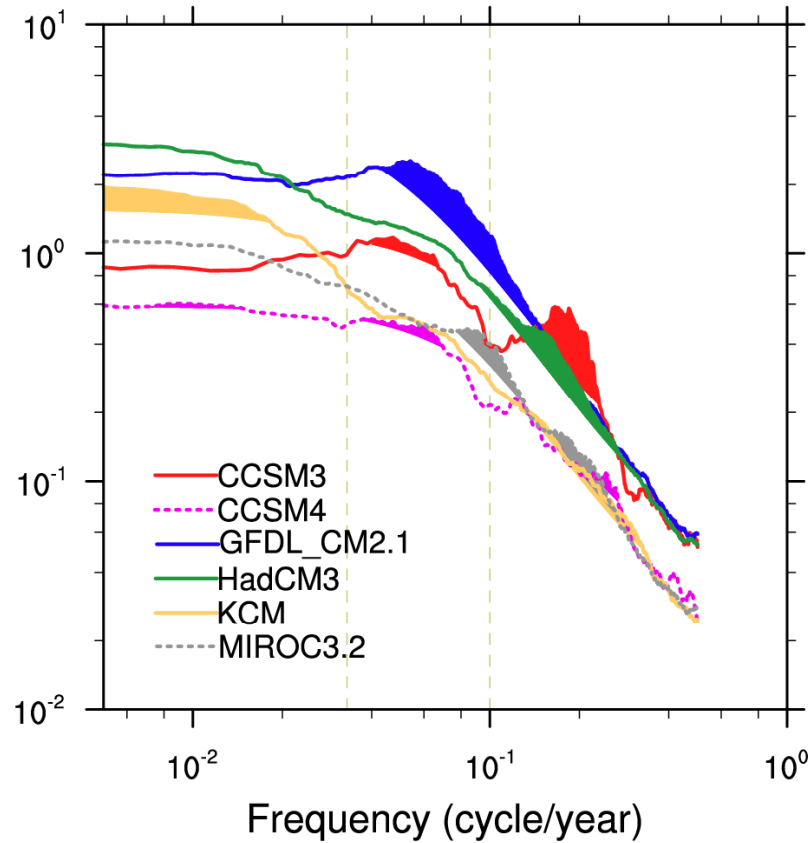
Predictability depends on many factors

- *Initial state dependence*
- *System-component dependence*
- *Time-scale dependence*
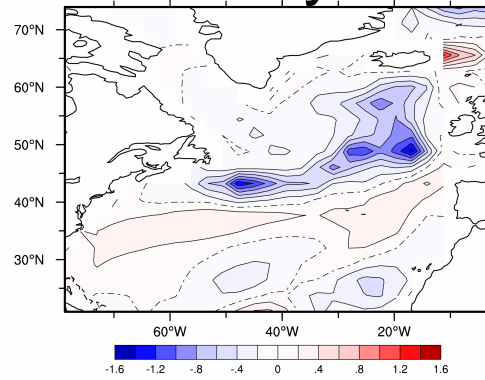- *Pattern dependence*
- *Model dependence*

Predictability depends on many factors

- *Initial state dependence*
- *System-component dependence*
- *Time-scale dependence*
- *Pattern dependence*
- *Model dependence*

Alternative methods for estimating initial value predictability

Predictability from Control Runs
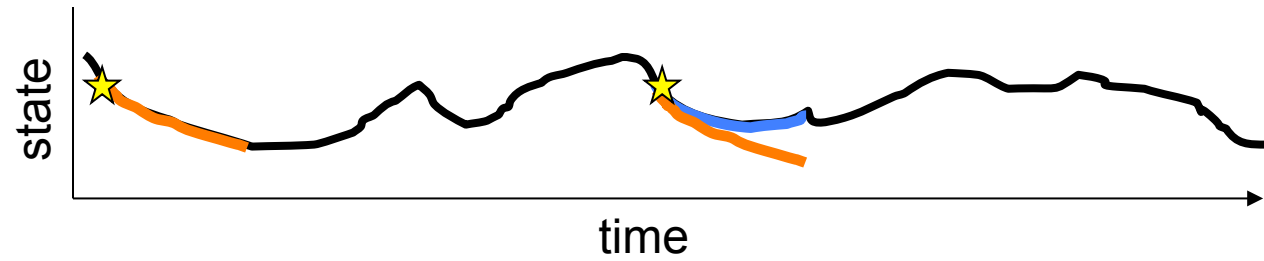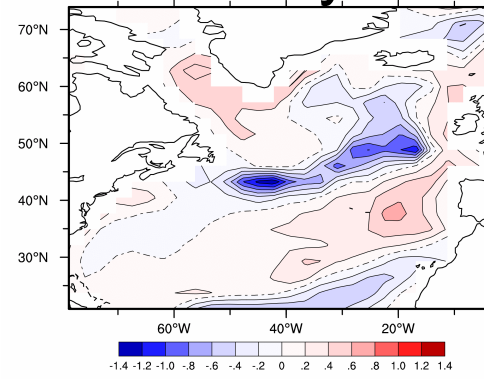
Predictability from Control Runs

Linear
dynamics

$$x(t) = \mathbf{C}(t - t_0)\mathbf{C}^{-1}(0)x(t_0)$$

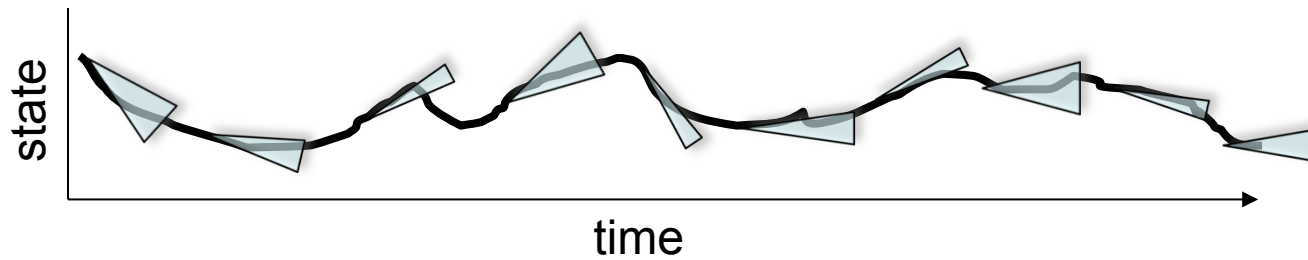$$\text{for } \mathbf{C}(\tau) = \text{lag cov matrix}$$

DelSole & Tippett (2009)
Lorenz (1969)

Attractor averages

state

time
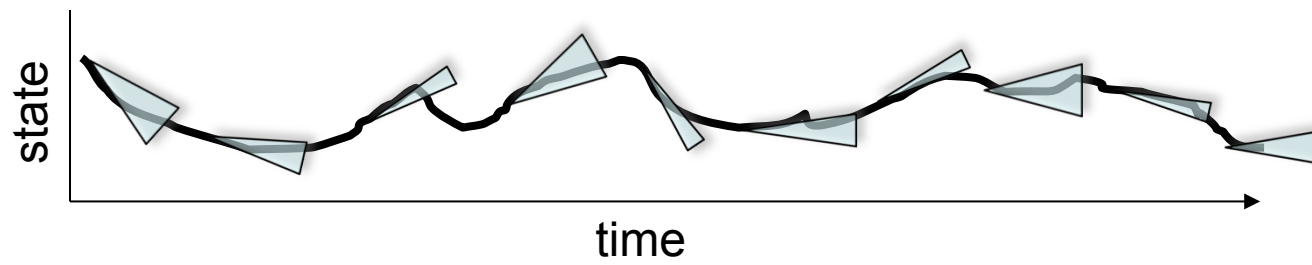
Attractor averages

Application to
Ocean Heat Content

Initial Value & Forced Predictability
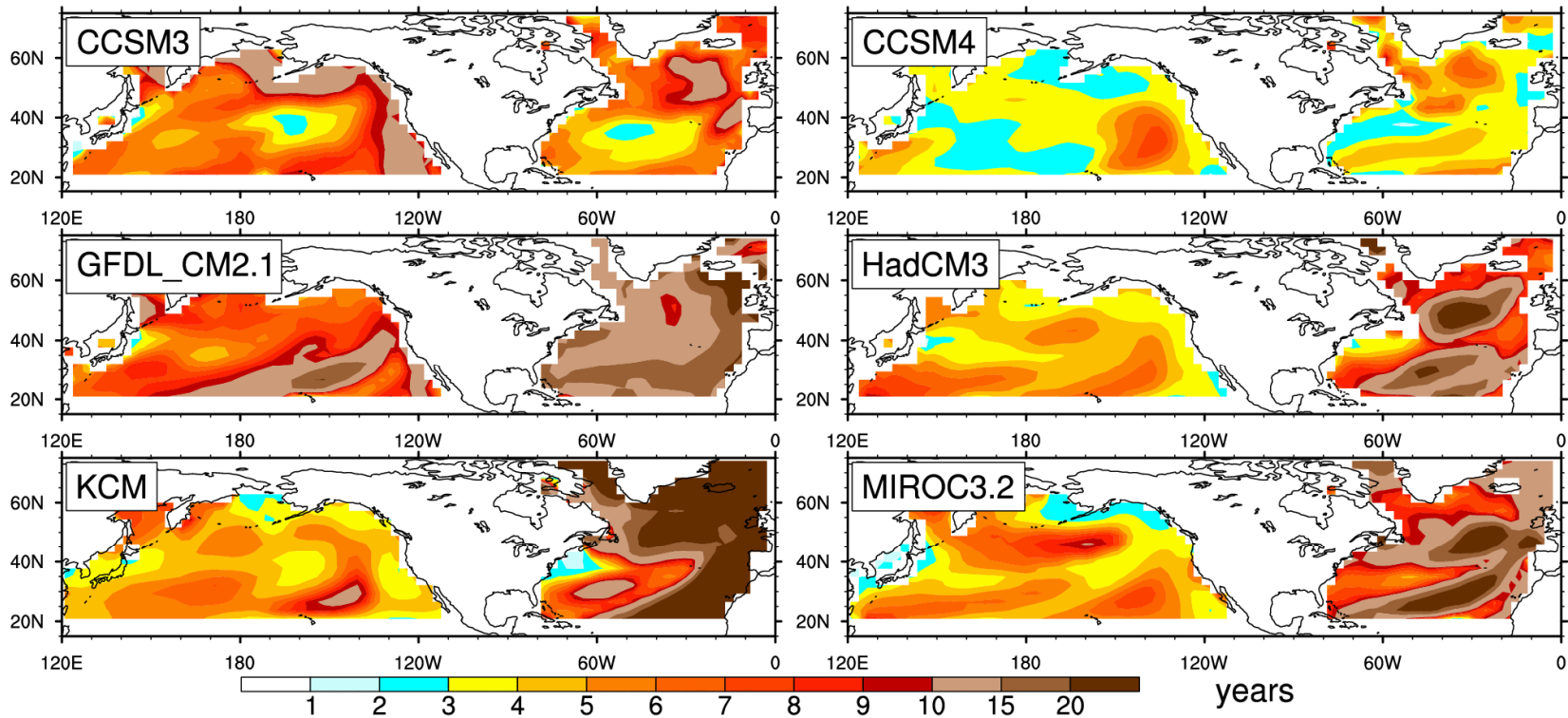*CMIP5*
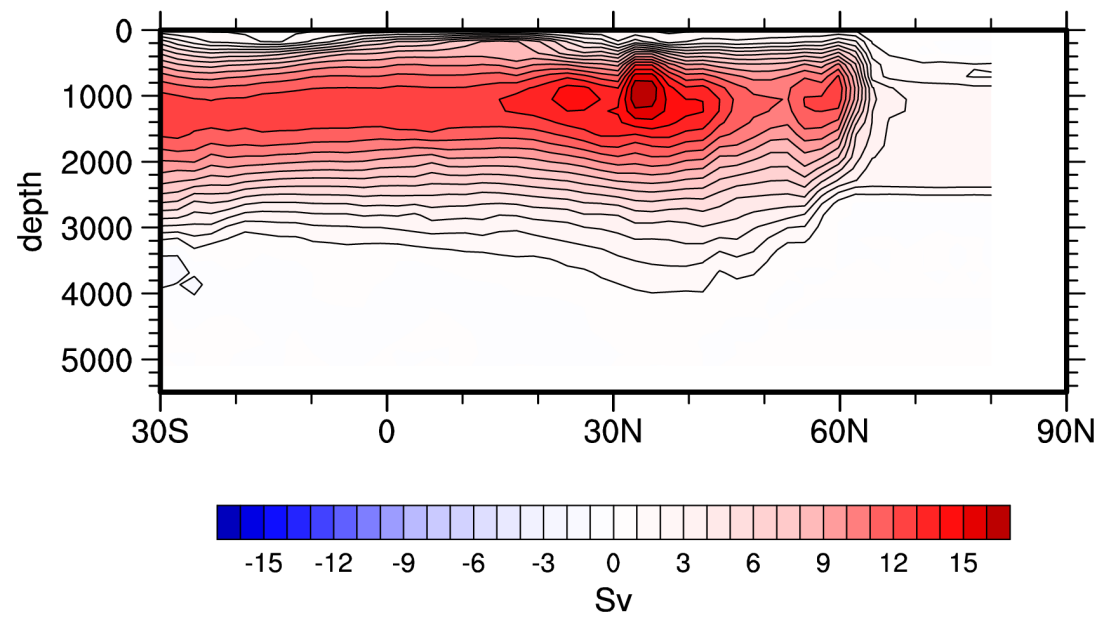*T0-300   10 PCs   RCP4.5*
Attractor average
North Atlantic
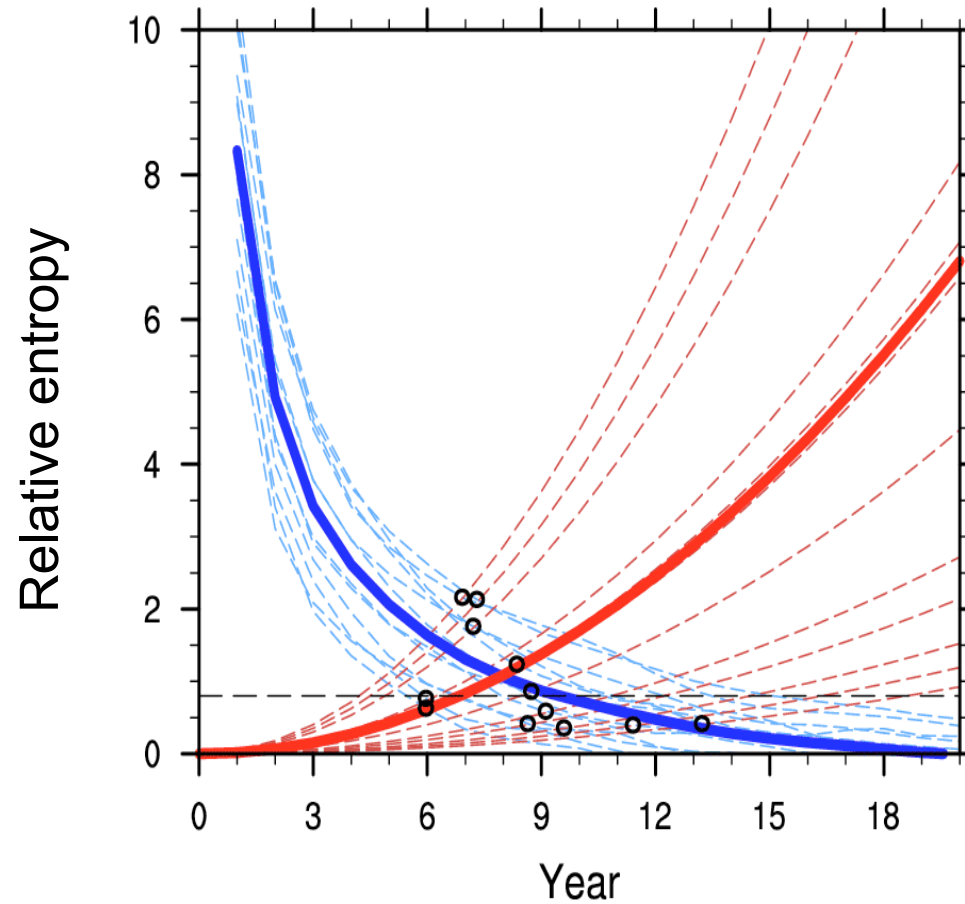
Year of Nominal Predictability Limit
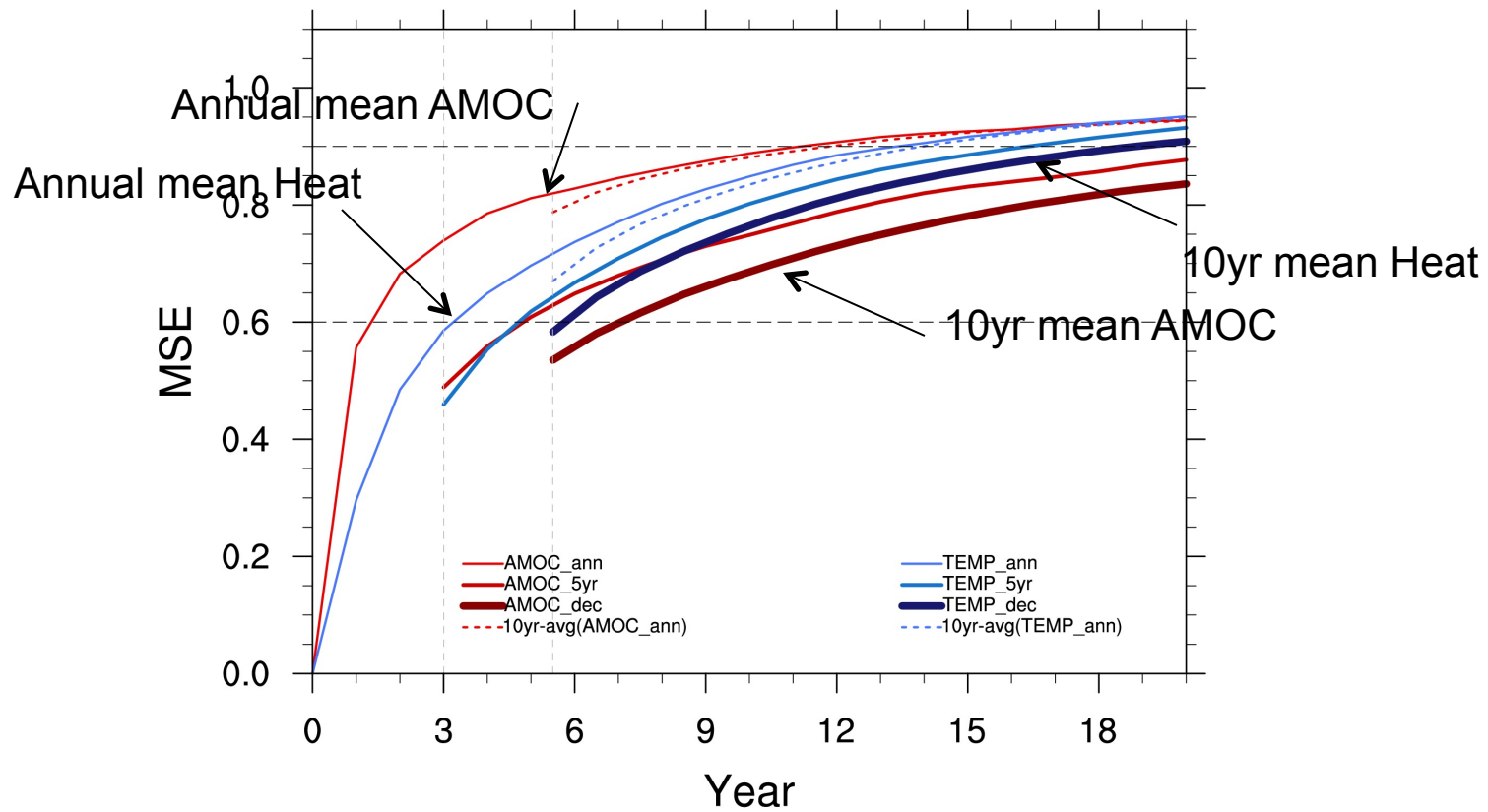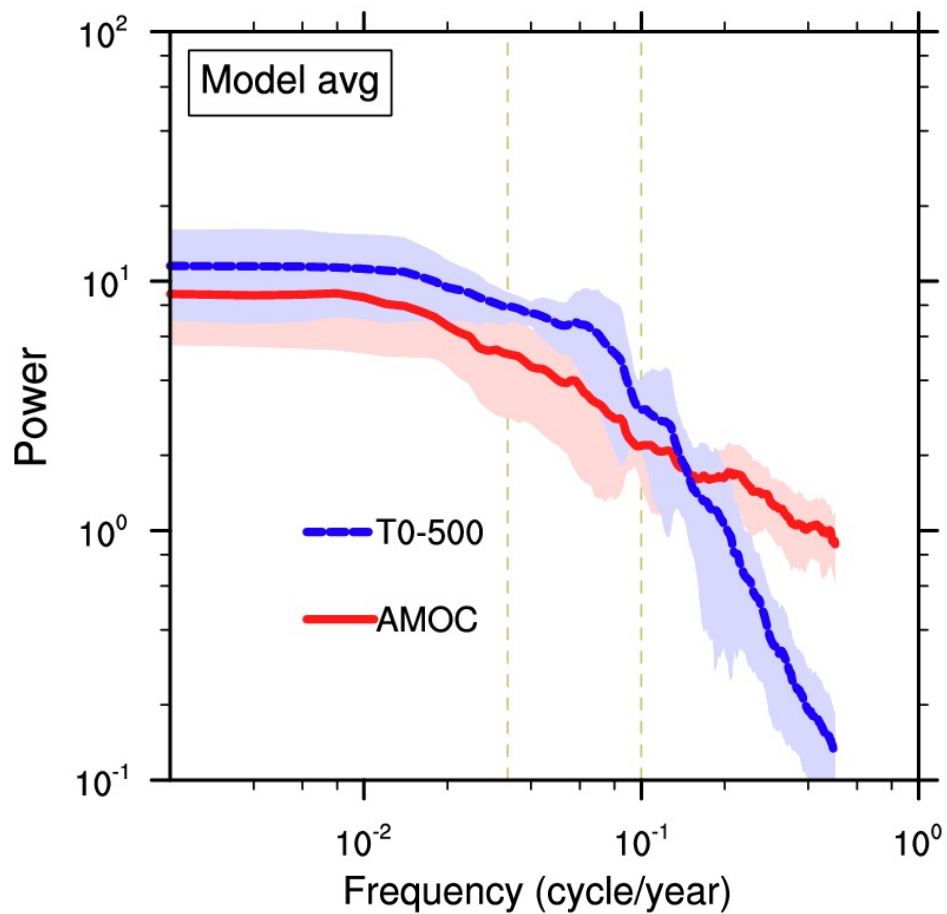
multivariate regression method

T0-300

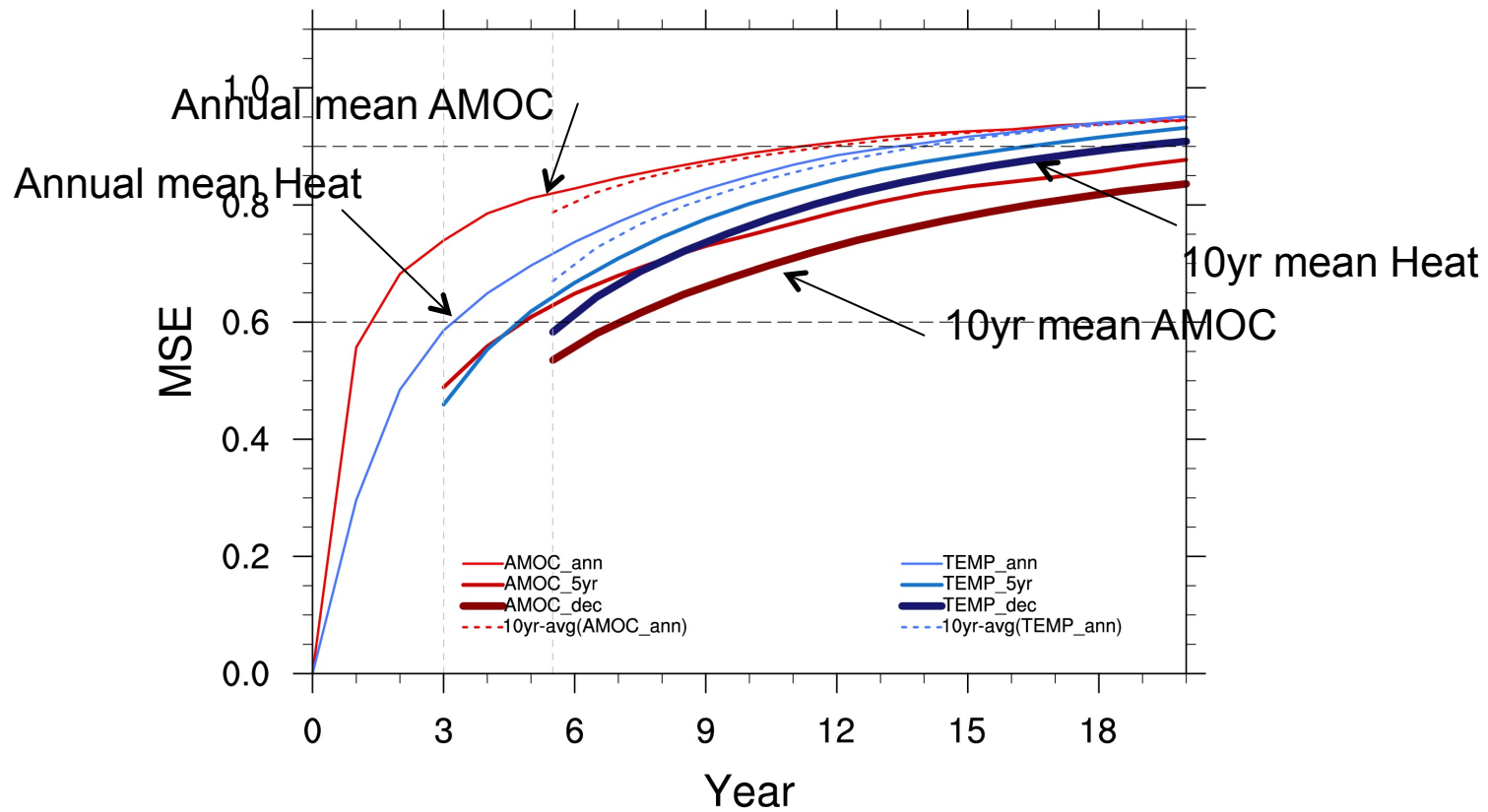# Comparing Predictability of AMOC and N Atl Heat Content

Predictability of 5- & 10-year means
CMIP5 Models

Average Power Spectra
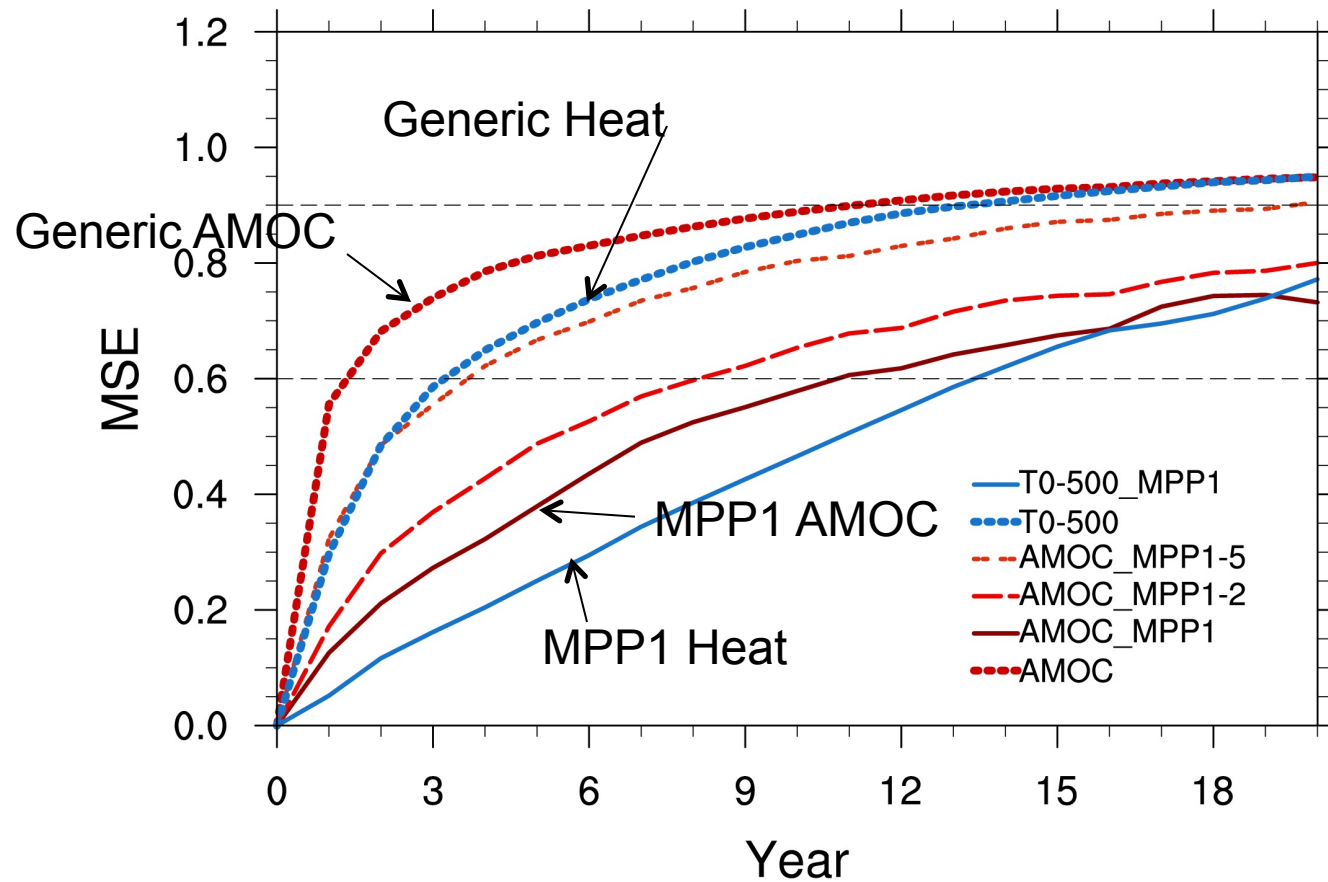# AMOC & N Atl Heat Content
9 CMIP5 models

Predictability of 5- & 10-year means
CMIP5 Models
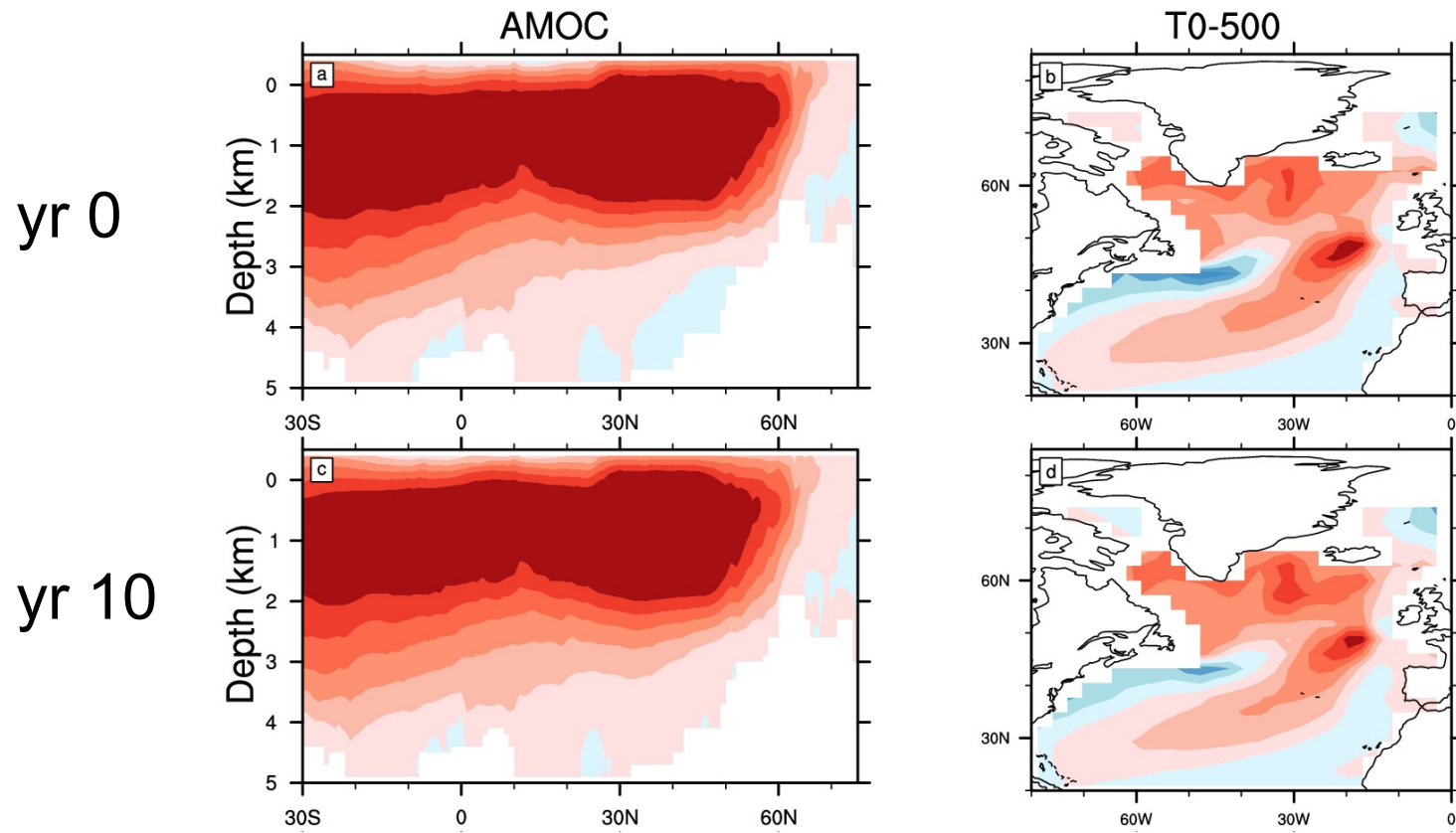
Are some components of AMOC especially predictable?

Predictability of Most Predictable Patterns
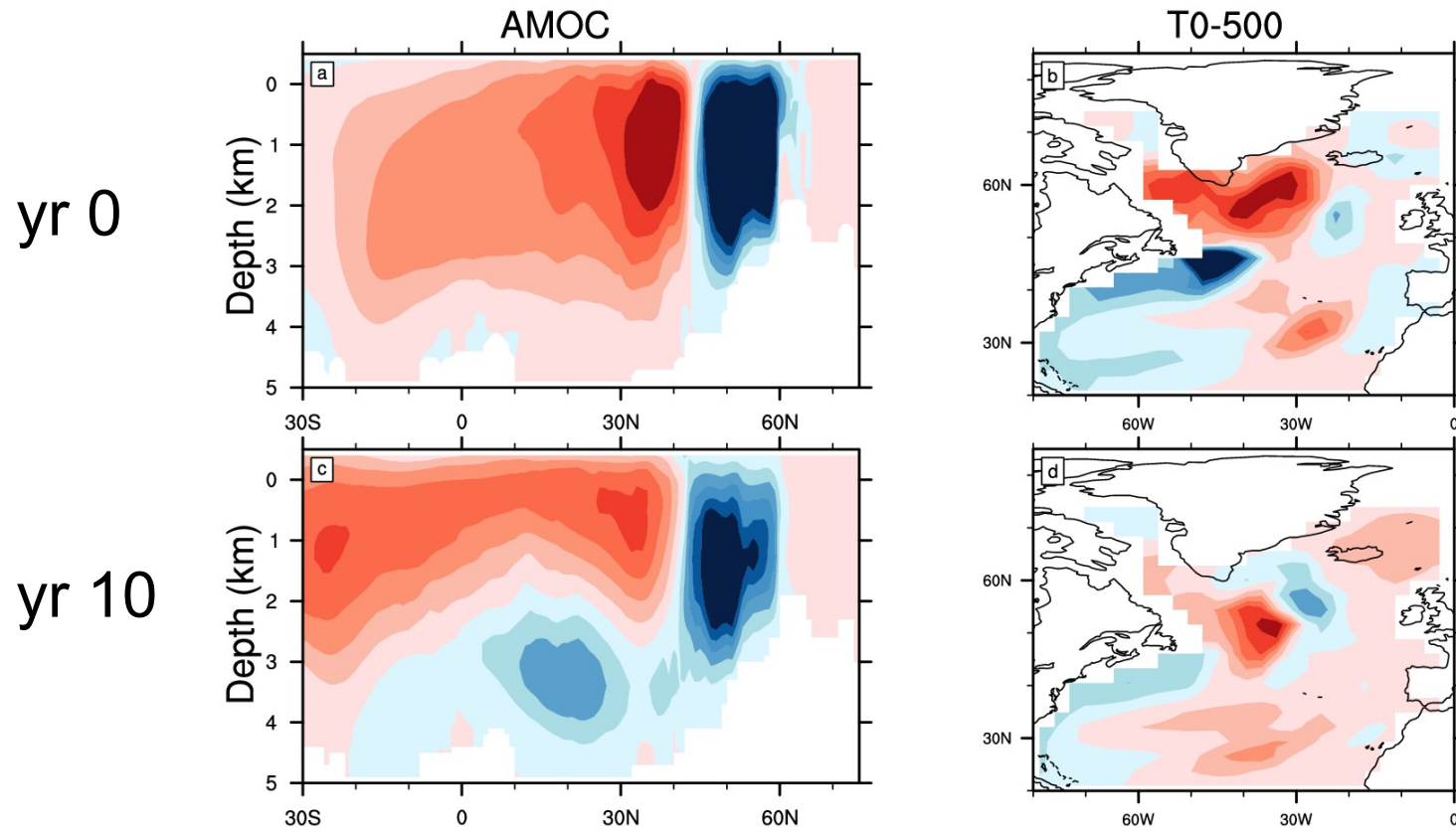
(canonical correlation analysis)*

Generic Heat

Generic AMOC

MPP1 AMOC

MPP1 Heat

Legend:
- T0-500_MPP1
- T0-500
- AMOC_MPP1-5
- AMOC_MPP1-2
- AMOC_MPP1
- AMOC

MSE (y-axis), Year (x-axis)

predictable component analysis (Renwick & Wallace, 1995)

Most Predictable AMOC Pattern
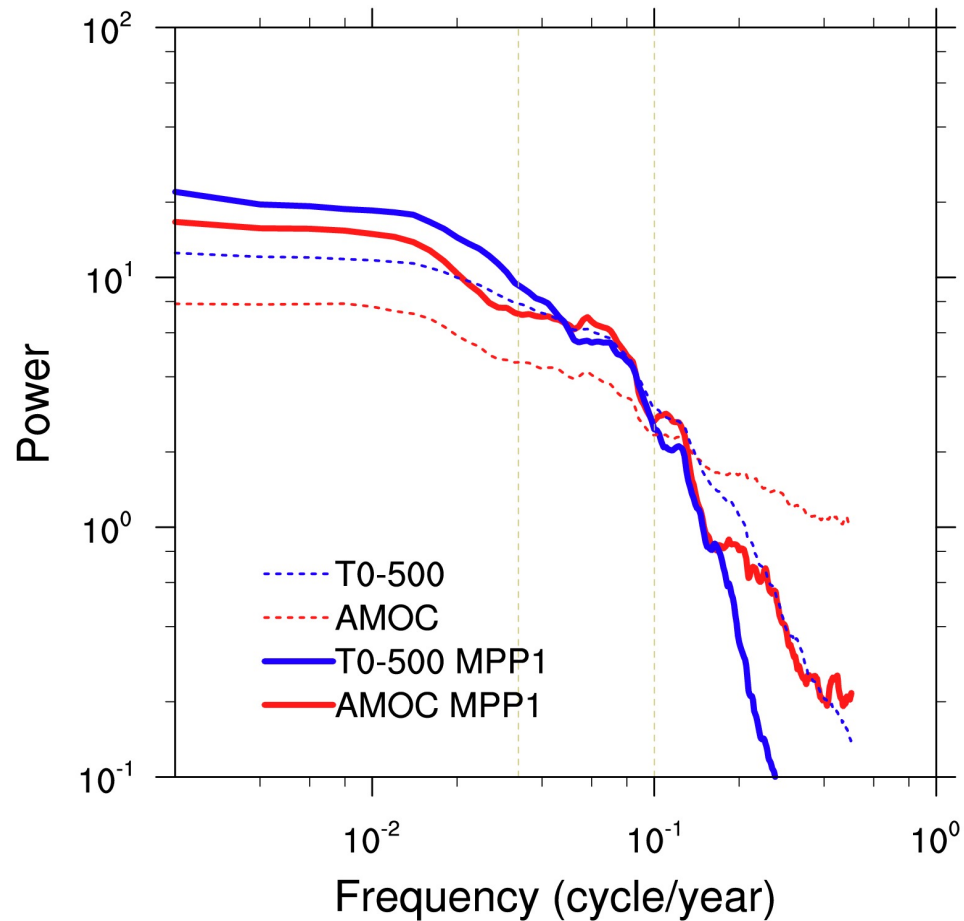## *CNRM-CM5*

Most Predictable AMOC Pattern
GFDL-CM3

Average Power Spectra
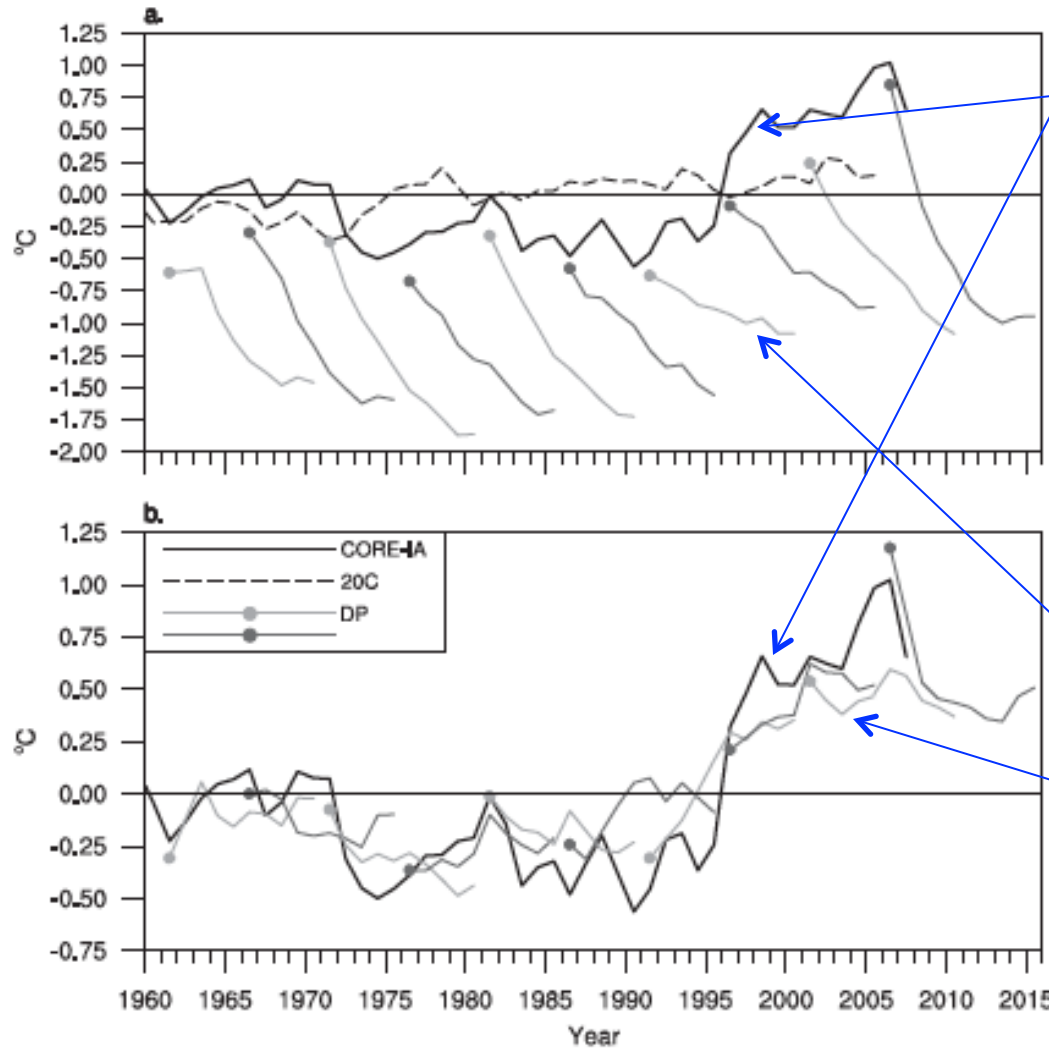of Most Predictable AMOC & Heat Content Patterns
9 CMIP5 models

Good news
- about 5-10 years of initial value predictability for OHC
- even more for
  - ✧ time averages
  - ✧ special patterns
  - ✧ certain initial conditions
  - ✧ certain regions

Bad news
- Models do not agree, so little is known about Nature's predictability

275m heat content in NAtl subpolar gyre

ocean model driven by
observed atmosphere
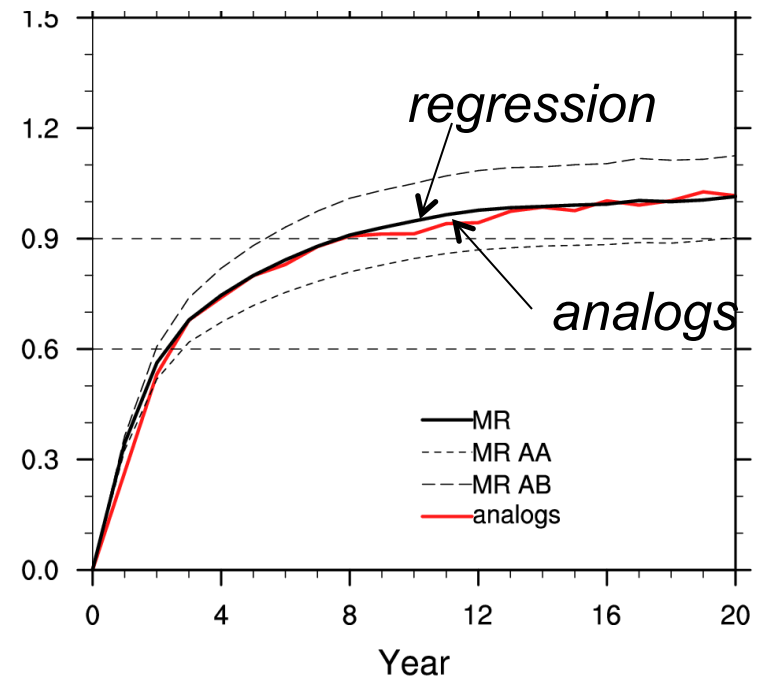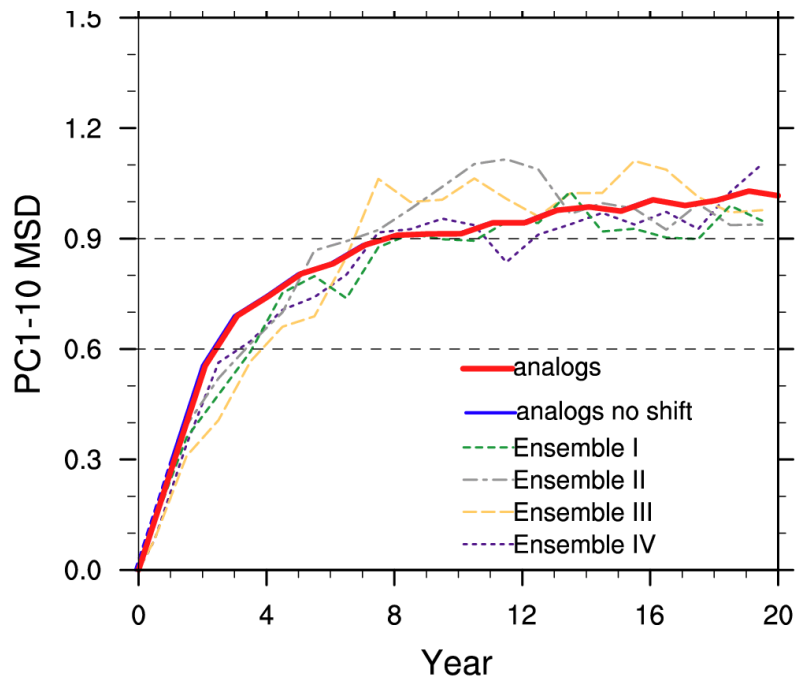
initialized coupled model

bias corrected

Yeager et al. 2012
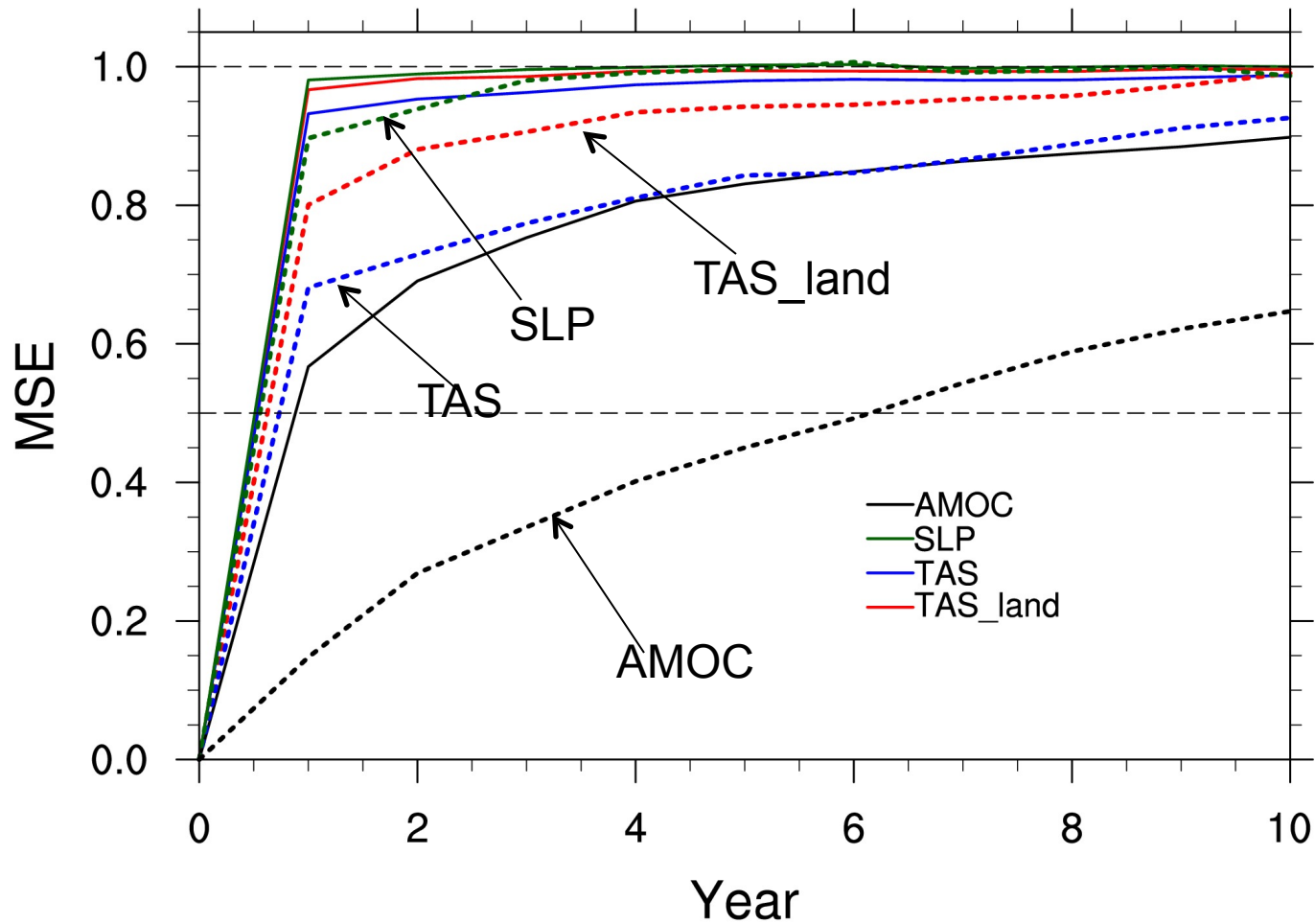
Predictability from
Ensembles, Analogs & Regression
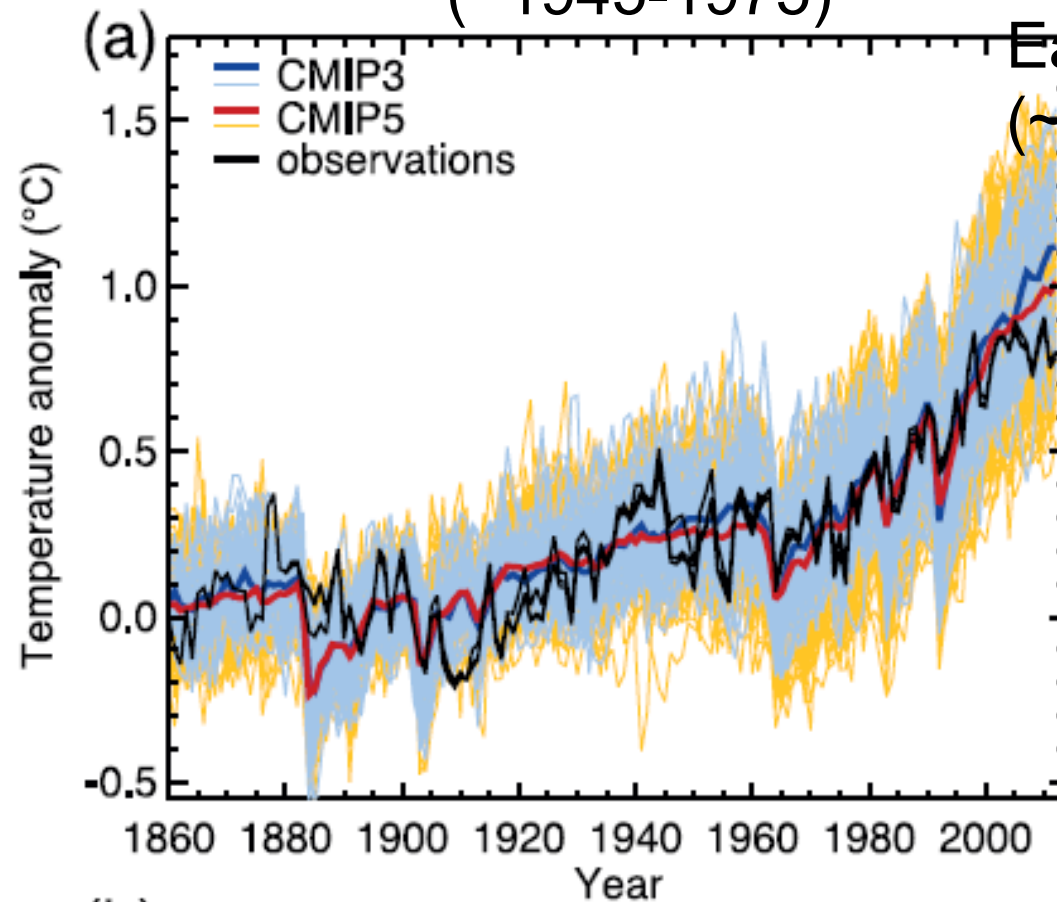## Attractor averages
CCSM3
North Pacific

Predictability of Most Predictable Patterns for AMOC, SLP$_{NH}$, TAS$_{NH}$, TAS$_{NH}$_land

(predictable component analysis)
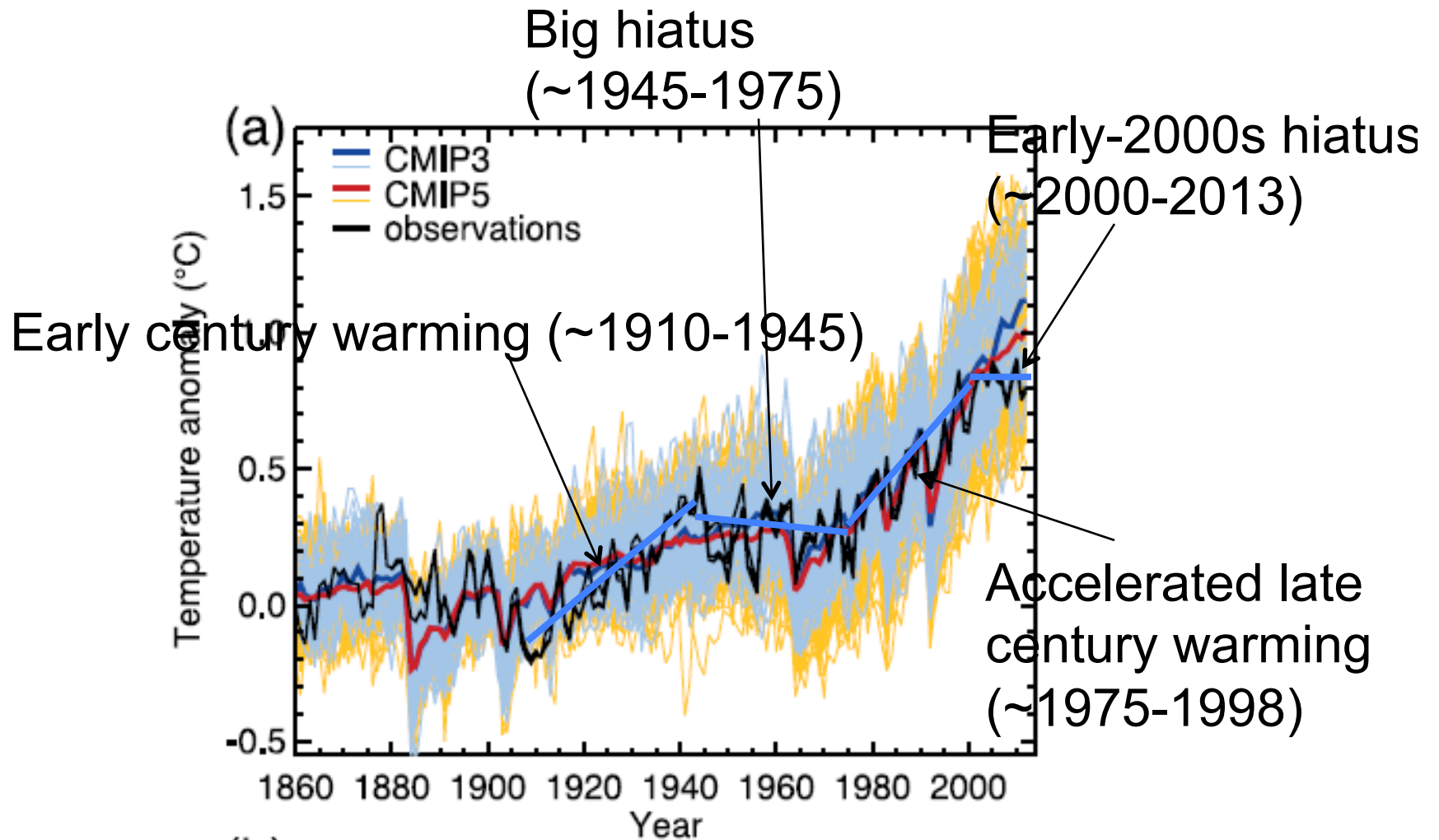
Global mean surface temperature

Big hiatus
(~1945-1975)

Early-2000s hiatus
(~2000-2013)

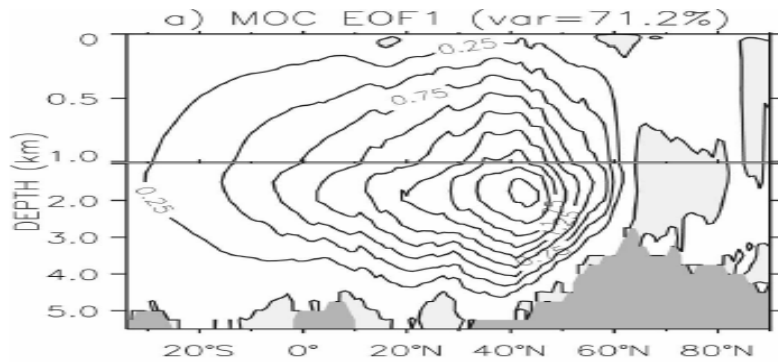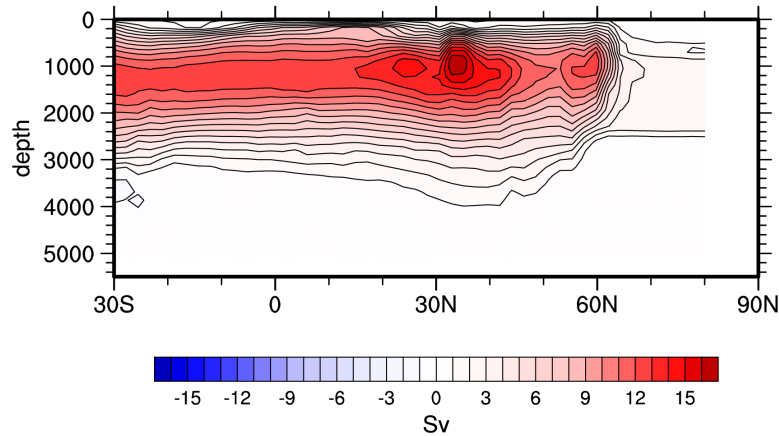(Bindoff, Stott, et al., 2013, IPCC AR5 ch. 10)

Global mean surface temperature
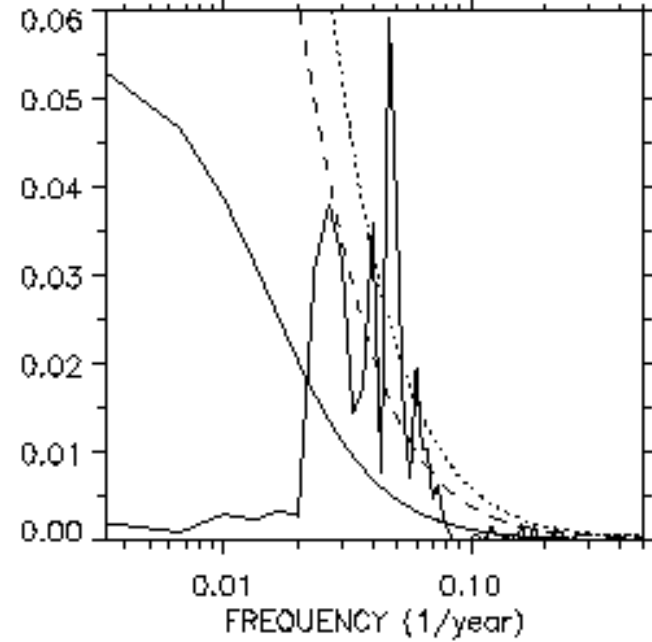
(Bindoff, Stott, et al., 2013, IPCC AR5 ch. 10)

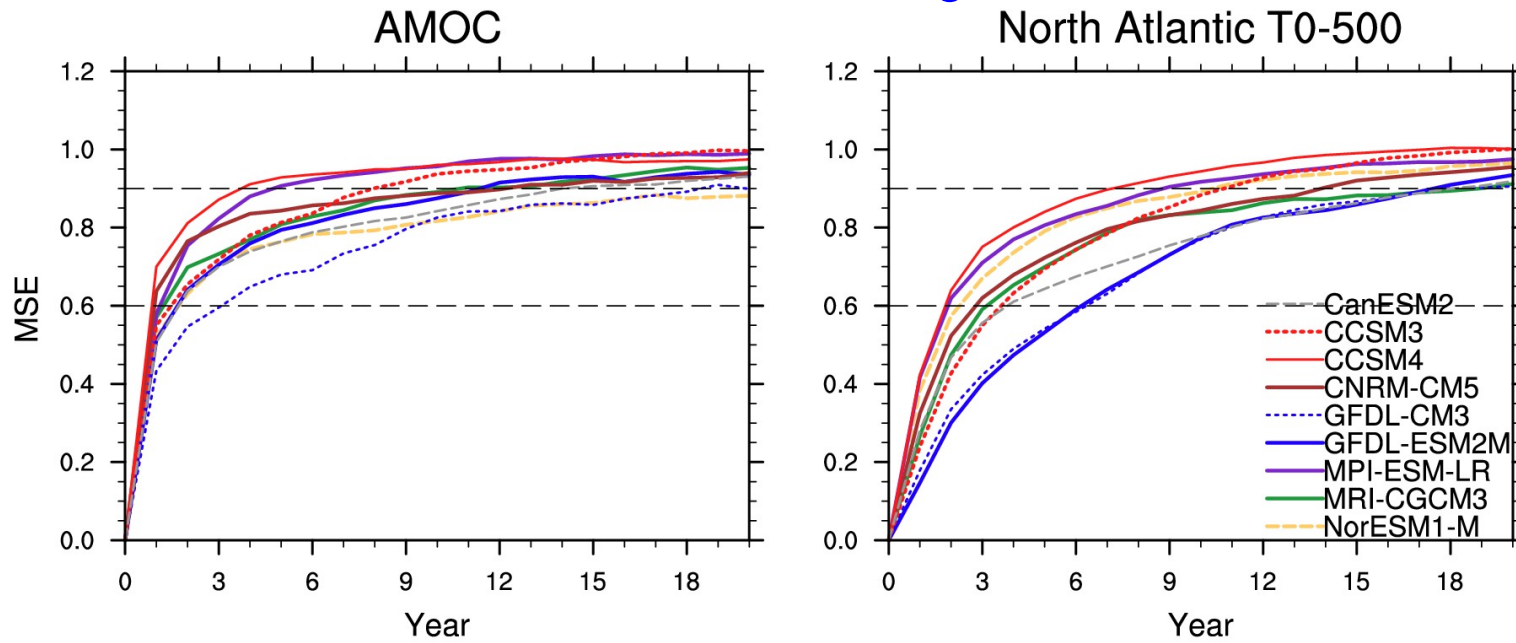# Predictability and Power Spectra

Peaks

CCSM3 AMOC EOF1

Danabasoglu (2008)

Predictability of CMIP5 Models
annual means

Attractor average

Branstator & Teng (2014)